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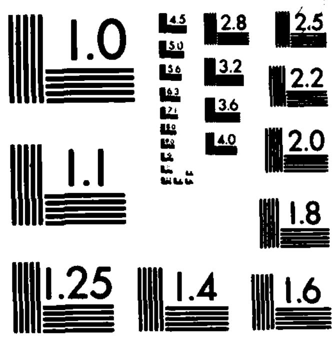

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BY

JOHN SPINELLI and MICHAEL A. STEPHENS

TECHNICAL REPORT NO. 336
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TESTS FOR EXPONENTIALITY WHEN ORIGIN AND SCALE PARAMETERS ARE UNKNOWN

By

John Spinelli and

Michael A. Stephens

1. INTRODUCTION

A test of exponentiality is a test of H_o; a given random sample of n values of x comes from the exponential distribution

$$(1) \qquad F(x) = 1 - \exp(-(x-A)/B), \qquad x > A ,$$

where B is a positive constant. This distribution will be referred to as $\exp(A,B)$, and A is the origin of the distribution. Throughout the paper we shall suppose the sample has been labelled in ascending order, so that $A < x_1 < x_2 < \cdots < x_n$.

Many tests of the above null hypothesis H_o have been proposed in the literature; in the great majority of these tests, B is unknown, and A is known. Then without loss of generality A can be assumed zero; if A is not zero the substitution $y_i = x_i - A$ produces a sample of ordered y-values from $\exp(0,B)$ when H_o is true. The parameter B is either estimated directly from the sample or is eliminated by exploiting connections between the exponential and the uniform distributions.

In this paper we discuss tests when both A and B are unknown. These tests have so far not been considered extensively, and a possible reason for this is because a simple device exists to change a test of

H_o with A unknown into a test with $A = 0$. This is to make the substitution $v_i = x_{i+1} - x_1$, $i = 1,\ldots,n-1$; it is a well known result that this substitution produces, on H_o, an ordered random sample of $n-1$ values of v from $\exp(0,B)$. Thus the test of H_o can be reduced to a test that the v_i come from $\exp(0,B)$, B unknown, using any of the many tests of this hypothesis that have been proposed. This method of dealing with the unknown A we shall call Method 1. Although Method 1 eliminates A, it may not necessarily give the most powerful test of H_o, and we therefore investigate two main alternative approaches, in which A is estimated directly rather than eliminated. The first of these alternatives, which we shall call Method 2, is to estimate A and B, and then to use tests based on the empirical distribution function (EDF). A second alternative, called Method 3, is to use a test based on a regression of the order statistics $x_1,\ldots,x_n$ on suitable constants along the horizontal axis. For example, the order statistics may be regressed against m_i where m_i is the expected value of the i-th order statistic from $\exp(0,1)$. On H_o, we have

$$(2) \qquad E(x_i) = A + Bm_i$$

where E denotes expectation, and suitable test statistics are measures of how well the order statistics fit the line (2). One such statistic, for example, is the Shapiro-Wilk (1972) statistic W_E, and other statistics recently proposed in the literature are measures of the correlation between x_i and m_i, or between x_i and statistics similar to m_i. Statistics based on these methods will be called regression statistics. In the next section the procedure is given for calculating EDF statistics; in section 3 regression statistics are described, and in section 4 we compare the methods.

2. EDF STATISTICS.

EDF statistics are statistics based on the discrepancy between the distribution (1) above, with estimates used for A and B, and the empirical distribution function of the sample of x-values. Many statistics have been proposed to measure this discrepancy, and we here concentrate on those which are usually called D^+, D^-, D, V, W^2, U^2 and A^2. The formal procedure for calculating these statistics is as follows:

(a) Calculate the following estimates of A and B:

$$\hat{B} = n(\bar{x}-x_1)/(n-1) \quad \text{and} \quad \hat{A} = x_1 - \hat{B}/n, \quad \text{where} \quad \bar{x} = \sum_{i=1}^{n} x_i/n.$$

(b) Calculate

$$w_i = (x_i-\hat{A})/\hat{B} \quad \text{and} \quad z_i = 1-\exp(-w_i), \quad \text{for} \quad i = 1,\dots,n.$$

(c) The test statistics are then given by the following formulas:

$$D^+ = \max_{1\le i\le n} [(i/n) - z_i]$$

$$D^- = \max_{1\le i\le n} [z_i - (i-1)/n]$$

$$D = \max(D^+,D^-)$$

$$V = D^+ + D^-$$

$$W^2 = \sum_{i=1}^{n} \left(z_i - \frac{2i-1}{2n}\right)^2 + \frac{1}{12n}$$

$$U^2 = W^2 - n\left(\bar{z} - \frac{1}{2}\right)^2, \quad \text{where} \quad \bar{z} = \sum_{i=1}^{n} z_i/n$$

$$A^2 = -\frac{1}{n}\sum_{i=1}^{n} (2i-1)(\ln z_i + \ln(1-z_{n+1-i})) - n.$$

The estimates $\hat{A}$ and $\hat{B}$ given above are derived from generalized least squares; they can be obtained also from the maximum likelihood estimates, modified to make them unbiased. The distributions of the test statistics, on H_o, will depend on n but not on the true values of A and B. These distributions are, for finite n, difficult to find, but asymptotic theory is available for the statistics W^2, U^2 and A^2. The asymptotic distributions are the same as those tabulated under the heading Case 4 by Stephens (1974, 1976); Case 4 refers to the situation in which A is zero and only B is estimated by maximum likelihood. The introduction of the new estimate $\hat{A}$, in addition to $\hat{B}$, does not effect the asymptotic distribution, because $\hat{A}$ has variance of the order $1/n^2$, and so is super-efficient in the notation of Darling (1955). However, for finite n there will be some difference in the null distributions, and these have been found by Monte Carlo methods. A list of upper tail percentage points is given in Tables 1 and 2. The slight difference in values of the asymptotic distributions for W^2, U^2 and A^2 represents an improvement in the calculations of these points since the appearance of Stephens (1974), although use of the earlier points makes only a very small difference in α-values. W^2 has also been examined by Van Soest (1969), who gave asymptotic theory and Monte Carlo points for $n = 10$ and 20. For $\sqrt{n}D^+$, $\sqrt{n}D^-$, and $\sqrt{n}D$ the asymptotic points have been found by extrapolating exact points given by Durbin (1975) for Case 4.

The test of fit therefore consists of calculating the appropriate statistic required, as described above, and referring either to Table 1 for direct percentage points, or calculating the modified form of the test statistic as given in Table 2 and referring to the asymptotic points only. The modifications are very useful when making a computer program for the tests.

Example. We illustrate the procedure on the set of data, kindly provided by Dr. W.G. Warren, listed in Table 3. There are $n = 32$ observations, measurements of modulus of rupture of wood beams. From the data we have $\bar{x} = 94.878$; then $\hat{B} = 53.356$ and $\hat{A} = 41.523$. The EDF statistics have values $\sqrt{n}D^+ = 1.115$, $\sqrt{n}D^- = 1.989$, $\sqrt{n}D = 1.989$, $\sqrt{n}V = 3.104$, $W^2 = 1.066$, $U^2 = 0.780$, $A^2 = 5.097$. Reference to Tables 1 and 2 show all these statistics to be highly significant and we reject the null hypothesis that the values are exponentially distributed.

Comment. In other goodness-of-fit tests based on the EDF, simple maximum likelihood estimators have usually been used, even when these are biased. In this case the maximum likelihood estimator of $\hat{A}$ would be x_1; the first value of w_i would then be 0, and the first value of z_i would be 0 also. This would produce an infinite value for the statistic A^2, so that this statistic could not be used. The change would of course also affect the Monte Carlo points for finite n for the other statistics, although the asymptotic distribution would remain unchanged. However, since A^2 has frequently been found to be a powerful statistic in goodness-of-fit, the unbiased estimate of A has been used and statistic A^2 is then available for test purposes.

3. REGRESSION METHODS

Suppose u_i, $i = 1,\ldots,n$ is an ordered random sample from $\exp(0,1)$ and let $m_i = E(u_i)$; then $E(x_i) = A+Bm_i$ as in (2) above. Test statistics of fit can then be based on how closely the data x_i fit this model. A possible test statistic is the correlation coefficient $r(x,m)$ calculated from the pairs (x_i,m_i), $i = 1,\ldots,n$. Strictly speaking this is not a correlation coefficient, since the m_i are constants and not random variables, but it has become customary to use this terminology since $r(x,m)$ is calculated from the usual formula for paired random variables, as follows.

Let $S_{xx} = \Sigma_i(x_i-\bar{x})^2$, $S_{xm} = \Sigma_i(x_i-\bar{x})(m_i-\bar{m})$ and $S_{mm} = \Sigma_i(m_i-\bar{m})^2$, where $\bar{x}$ and $\bar{m}$ are the means of the x-values and the m-values respectively. Then $r^2(x,m) = S^2_{xm}/(S_{xx}S_{mm})$. An alternative method of plotting is to put $h_i = -\ln\{1-i/(n+1)\}$ along the x-*axis*, *instead* of m_i; h_i is a good approximation to m_i and is easier to calculate. The corresponding correlation coefficient will be called $r(x,h)$. For both these statistics a high value (that is, approaching 1) will suggest a good fit to the exponential model.

It is common in regression to set up an Analysis of Variance table with SS denoting Sum of Squares:

Source of variation	
Regression SS	$= S^2_{xm}/S_{mm}$
Error SS	$= S_{xx}-S^2_{xm}/S_{mm}$
Total SS	$= S_{xx}$

The Error SS may also be written $\sum_i (x_i - \hat{x}_i)^2$, where $\hat{x}_i = \hat{A} + \hat{B}m_i$ and is clearly also a measure of how well the data fits the estimated line (2); in fact Error SS/Total SS = $T_m = 1-r^2(x,m)$ can be used as the test statistic, with high values of T_m leading to rejection of H_o. The parallel statistic $T_h = 1-r^2(x,h)$ could be used also; some critical values for use with censored samples have been given by Smith and Bain (1976, Table III). These were found by Monte Carlo sampling. In Table 4 of this paper are given critical values for T_m, and further values for T_h. These are based on Monte Carlo samples, using 10,000 samples for each n. For Table III of Smith and Bain (1976) the number of Monte Carlo samples was 100,000/n, so that the values in Table 4 for $n > 10$ might be expected to be more accurate.

Example. For the example quoted in Section 2, we have $r^2(x,m) = 0.640$ and $r^2(x,h) = 0.611$; the corresponding values of T_m and T_h are 0.360 and 0.389. Reference to Table 4 shows these statistics to be clearly sig ficant at level 0.01; thus the hypothesis that these data are exponential is rejected.

An alternative method of measuring the goodness-of-fit was introduced by Shapiro and Wilk (1972), and consists of comparing the generalized least squares estimate of B given from the linear model (2) with the estimate obtained from the sample variance. The resulting test statistic W_E is given by

$$W_E = \frac{n(\bar{x}-x_{(1)})^2}{(n-1)S_{xx}^2} .$$

Tables of percentage points for W_E, for n from 3 to 100, have been given by Shapiro and Wilk (1972). The test is a two-tailed test. The value of W_E is 0.1742, and reference to Table 1 of Shapiro and Wilk (1972) shows this to be highly significant in the lower tail, so again H_0 will be rejected.

There is a weakness in connection with W_E. It is possible to find other distributions for which the limit of $\{\bar{x}-x_{(1)}\}/S_{xx}$, as $n\to\infty$, is the same as that for the exponential, i.e. 1, and the test based on W_E will not necessarily be consistent; this was noted by Sarkadi (1975). A distribution with this property, for example, is the Beta(a,b) distribution, with $a < 1$ and $b = a(a+1)/(1-a)$; two pairs (a,b) are $(1/4,5/12)$ and $(1/2,3/2)$. Monte Carlo Samples of size n were taken from Beta(1/4,5/12) and the percentage of 1000 samples rejected by W_E in a test for exponentiality at the 10% level were recorded. The results were, for $n = 10$, 15.8%; for $n = 20$, 9.4%; for $n = 50$, 7.2%; and for $n = 100$, 2.2%. Corresponding percentages for the statistic A^2 were 61.9%, 92.4%, 100.0% 100.0%. For the same sample sizes, and for samples from the Beta (1/2,3/2) distribution, the percentages rejected were, for W_E, 7.7, 5.8, 4.4, 1.5; and for A^2, 22.1, 38.2, 73.1, 96.4. The inconsistency of W_E is thus apparent. In contrast, Gerlach (1979) has shown that $r^2(x,m)$ (and hence T_m) always gives a consistent test and this would appear to be true for $r^2(x,h)$ and T_h also.

4. POWER STUDIES

The efficiencies of the above methods of dealing with unknown A have been examined by extensive Monte Carlo studies. Random spaces of size n were taken from a broad range of distributions including especially those distributions, such as the Weibull, gamma and lognormal distributions, which are used in the literature as practical alternatives to the exponential in models for reliability studies or renewal processes. These were then tested for exponentiality using the several methods described above, and tables prepared showing the percentage of samples declared significant when the test was of size α. It was noted by Stephens (1974) and again by Dyer (1974) that knowledge of the exact values of parameters (in the context of testing for normality) does not always help in deciding on the distributional form; the power of the test against densities of different shape is greater when the statistician estimates his own parameters. For this reason, tables were prepared to compare the results for A unknown, with those obtained when A was assumed known (Case 4 above). Powers for A known are given first, in Table 5. They agree within sampling fluctuations with previous values given in an unpublished technical report by Stephens (1978). Table 6 shows the results for Method 1 (subtracting the minimum and treating as Case 4), Table 7 gives results for EDF statistics with A and B estimated and Table 8 compares the best of the EDF statistics with the Shapiro-Wilk W_E and with the correlation statistics T_m and T_h, respectively equivalent to $r^2(x,m)$ and $r^2(x,h)$. Van Soest (1969) also gives some power studies against Gamma alternatives, and our results agree where they overlap.

Comments. (a) There is, overall, a reduction of power for EDF statistics when A must be estimated, in contrast to the situation in testing for normality using these statistics (Stephens, 1974; Dyer, 1974).

(b) Of the EDF statistics, A^2 is best on the whole, for both methods 1 and 3, though W^2 is close behind. D^+ or D^- will usually be more powerful than A^2, but their limitation is that one must know which of the two should be used.

(c) In a comparison of methods 1 and 2, both of which use EDF statistics, A^2 with method 2 is overall slightly better than A^2 with method 1. When there is a very high probability of small values (e.g. for χ_1^2, or Weibull (0.5)), method 1 is better. These differences extend to the other statistics also.

(d) From Table 8, where A^2 and W^2 with method 2 are compared with the regression statistics, it can be seen that the simple regression statistics T_m and T_h are poor in power, particularly for large sample sizes. T_h is mostly worse than T_m.

(e) Statistics W_E, and W^2 or A^2, are competitive with each other, with sometimes the former statistic and sometimes the latter pair being more powerful. There are indications that for some alternatives, e.g. *Beta (1,4), the half-normal* and the log-normal distributions, W_E improves with large samples. But the serious drawback that W_E is inconsistent, and so will rarely detect certain distributions even with large samples, will put W_E out of favour with many statisticians.

(f) If D^+, D^-, or W_E are to be used, some indication of the nature of the alternative would have to be available. Of course, D^+ and D^- can both be calculated, and this is a useful general procedure towards an analysis of the data; but if then the most significant statistic is chosen for a test, there will be an unknown change in the true significance level. In view of these reservations involving the other statistics, our overall recommendation would therefore be to use method 2 with A^2, as described in Section 2, for an overall omnibus statistic for this problem.

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TABLE 1

Upper tail percentage points for $\sqrt{n}D^+$, $\sqrt{n}D^-$, $\sqrt{n}D$, $\sqrt{n}V$, W^2, U^2, and A^2, for a test of exponentiality with location and scale parameters estimated as in Section 2.

a) Statistic $\sqrt{n}D^+$

	Upper tail significance level α					
	.25	.15	.10	.05	.025	.01
5	.491	.569	.639	.743	.825	.917
10	.580	.674	.745	.851	.952	1.038
15	.610	.700	.768	.872	.978	1.077
20	.624	.716	.785	.894	.995	1.108
25	.635	.725	.799	.909	1.010	1.125
50	.660	.758	.832	.943	1.051	1.163
100	.682	.778	.853	.967	1.074	1.189
∞	.723	.820	.886	.996	1.094	1.211

b) Statistic $\sqrt{n}D^-$

5	.627	.705	.753	.821	.891	.955
10	.671	.761	.825	.916	.993	1.089
15	.688	.783	.842	.933	1.022	1.111
20	.696	.791	.855	.949	1.041	1.132
25	.702	.795	.860	.958	1.052	1.149
50	.710	.807	.874	.976	1.072	1.178
100	.717	.814	.879	.984	1.089	1.192
∞	.723	.820	.886	.996	1.094	1.211

c) Statistic $\sqrt{n}D$

5	.683	.749	.793	.865	.921	.992
10	.753	.833	.889	.977	1.048	1.119
15	.771	.865	.912	1.002	1.079	1.163
20	.786	.872	.927	1.021	1.099	1.198
25	.792	.878	.936	1.033	1.115	1.215
50	.813	.879	.960	1.061	1.149	1.257
100	.824	.911	.972	1.072	1.171	1.278
∞	.840	.927	.995	1.094	1.184	1.298

TABLE 1 Continued

	.25	.15	.10	.05	.025	.01
d) Statistic $\sqrt{n}V$						
5	1.098	1.186	1.234	1.314	1.400	1.494
10	1.194	1.294	1.363	1.461	1.556	1.662
15	1.225	1.325	1.392	1.504	1.596	1.701
20	1.245	1.346	1.419	1.536	1.635	1.769
25	1.260	1.366	1.438	1.559	1.658	1.796
50	1.292	1.400	1.481	1.600	1.701	1.847
100	1.310	1.419	1.502	1.647	1.740	1.897
∞	1.334	1.444	1.532	1.656	1.770	1.910
e) Statistic W^2						
5	.083	.102	.117	.141	.166	.197
10	.097	.122	.142	.176	.211	.259
15	.103	.130	.151	.188	.229	.281
20	.106	.133	.157	.195	.237	.293
25	.107	.135	.160	.199	.247	.301
50	.111	.141	.166	.209	.256	.319
100	.113	.144	.170	.215	.263	.328
∞	.116	.148	.175	.222	.271	.338
f) Statistic U^2						
5	.068	.083	.093	.113	.131	.153
10	.075	.094	.108	.131	.155	.187
15	.080	.099	.114	.139	.165	.200
20	.082	.102	.117	.143	.170	.207
25	.083	.104	.119	.146	.173	.212
50	.087	.108	.124	.152	.180	.223
100	.089	.110	.126	.155	.184	.229
∞	.090	.112	.129	.159	.189	.236
g) Statistic A^2						
5	.460	.555	.621	.725	.848	.989
10	.545	.660	.747	.920	1.068	1.352
15	.575	.720	.816	1.009	1.198	1.495
20	.608	.757	.861	1.062	1.267	1.580
25	.625	.784	.890	1.097	1.317	1.635
50	.680	.838	.965	1.197	1.440	1.775
100	.710	.875	1.008	1.250	1.510	1.855
∞	.736	.916	1.062	1.321	1.591	1.959

TABLE 2

Modifications and percentage points for the modified statistics W^2, U^2, and A^2.

Statistic	Modification	Upper tail significance level α .25	.15	.10	.05	.025	.01
W^2	$W^2(1 + {}^{2.8}/n - {}^{3}/n^2)$	.116	.148	.175	.222	.271	.338
U^2	$U^2(1 + {}^{2.3}/n - {}^{3}/n^2)$	.090	.112	.129	.159	.189	.230
A^2	$A^2(1 + {}^{5.4}/n - {}^{11}/n^2)$	.736	.916	1.062	1.321	1.591	1.959

TABLE 3

Thirty-two ordered values of modulus of rupture for Douglas Fir and Larch two-by-fours.

43.19	49.44	51.55	55.37	56.63	67.27	78.47	86.59
90.63	92.45	94.24	94.35	94.38	98.21	98.39	99.74
100.22	103.48	105.54	105.54	107.13	108.14	108.64	108.94
109.62	110.81	112.75	113.64	116.39	119.46	120.33	131.57

Values and significance levels of the regression and EDF test statistics for exponentiality calculated on the above data.

Statistic	value	significance level α
$\sqrt{n}D^+$	1.115	<.025
$\sqrt{n}D^-$	1.989	<.01
$\sqrt{n}D$	1.989	<.01
$\sqrt{n}V$	3.104	<.01
W^2	1.066	<.01
U^2	0.780	<.01
A^2	5.097	<.01
$r^2(x,m)$	0.640	<.01
$r^2(x,h)$	0.611	<.01
W_E	0.174	<.01

TABLE 4

Percentage points for $T_m = 1-r^2(x,m)$ and for $T_h = 1-r^2(x,h)$

Statistic T_m

n	Upper tail α-level .25	.15	.10	.05	.025	.01
5	.140	.178	.201	.261	.328	.392
10	.104	.133	.156	.192	.232	.272
15	.087	.110	.129	.163	.194	.232
20	.076	.096	.113	.141	.169	.210
25	.065	.083	.097	.123	.152	.189
50	.044	.056	.067	.085	.108	.143
100	.028	.036	.043	.057	.074	.105
∞	.000	.000	.000	.000	.000	.000

Statistic T_h

n	.25	.15	.10	.05	.025	.01
5	.136	.167	.192	.235	.285	.352
10	.102	.129	.154	.199	.234	.285
15	.084	.110	.132	.170	.207	.259
20	.073	.097	.116	.152	.191	.241
25	.064	.084	.102	.135	.175	.223
50	.044	.060	.073	.101	.133	.176
100	.029	.040	.050	.069	.094	.129
∞	.000	.000	.000	.000	.000	.000

TABLE 5

Power Comparisons: A known

The table shows the percentage of 2500 samples declared significant by the statistic, when the test used was a 10% test.

Alternative distr.	Sample size	D^+	D^-	D	V	W^2	U^2	A^2
Beta (1,4)	10	3.6	16.8	11.7	11.4	13.0	13.5	10.6
	20	2.5	22.7	15.4	14.4	17.7	15.7	14.4
	50	1.1	39.5	27.4	23.6	31.2	24.4	27.8
Chi-square 1	10	54.0	1.7	33.6	30.9	38.1	33.6	56.2
	20	72.2	0.7	57.2	50.4	62.6	53.8	71.7
	50	96.5	1.0	91.4	87.3	94.5	89.0	98.3
Chi-square 4	10	1.2	43.3	32.5	29.7	37.9	34.4	31.7
	20	1.3	68.7	56.1	50.8	63.7	55.5	61.6
	50	4.5	95.8	90.5	86.7	95.0	90.6	96.3
Half-cauchy	10	53.9	2.8	42.7	36.7	45.1	37.8	46.0
	20	75.8	0.8	65.6	57.7	68.1	59.2	68.5
	50	95.4	0.0	92.8	88.5	94.2	89.1	93.9
Half-normal	10	1.6	26.2	19.7	19.0	21.2	20.4	17.0
	20	0.8	38.6	27.0	24.8	31.7	27.2	27.4
	50	1.0	66.5	54.4	48.7	62.1	51.6	58.8
Log-normal(1)	10	16.0	14.4	16.6	16.3	17.4	17.5	14.9
	20	22.5	15.6	22.5	23.2	25.6	26.5	24.8
	50	36.0	23.8	36.7	48.4	43.3	51.2	50.5
Log-norm(2.4)	10	91.2	0.2	83.2	78.3	86.0	79.8	89.8
	20	99.6	0.0	98.4	97.0	99.1	97.0	99.4
	50	100.0	0.1	100.0	100.0	100.0	100.0	100.0
Uniform	10	3.4	53.1	42.2	47.9	51.5	48.0	44.7
	20	17.6	80.0	69.0	78.5	81.2	74.7	77.5
	50	84.9	98.8	97.0	99.7	99.4	98.6	99.8
Weibull (0.5)	10	79.4	0.3	63.8	57.1	68.9	59.1	81.1
	20	95.2	0.0	91.0	84.1	92.2	86.2	96.8
	50	100.0	0.5	100.0	99.9	100.0	99.9	100.0
Weibull (2.0)	10	2.3	75.7	65.8	64.1	76.2	69.3	69.6
	20	9.8	96.4	91.8	91.8	97.0	93.8	96.6
	50	49.8	100.0	100.0	100.0	100.0	100.0	100.0

TABLE 6

Power Comparisons using Method 1 (subtracting X_1 from all others).

The table shows the percentage of 2500 samples declared significant by the statistic, when the test used was a 10% test.

Alternative distr.	Sample size	D^+	D^-	D	V	W^2	U^2	A^2
Beta (1,4)	10	4.4	16.8	12.5	12.0	13.8	13.1	11.6
	20	2.9	22.4	14.8	14.6	17.4	15.0	14.8
	50	1.2	38.7	27.0	23.0	31.0	24.0	27.0
Chi-square 1	10	40.3	2.5	23.0	21.6	25.3	22.2	36.6
	20	63.2	1.0	46.9	40.7	51.2	44.4	64.1
	50	94.4	1.0	87.3	81.6	91.3	84.9	96.0
Chi-square 4	10	2.6	24.8	18.2	16.4	20.4	18.4	15.6
	20	1.4	42.1	31.0	28.9	35.0	30.6	31.7
	50	2.2	78.0	68.0	63.5	74.5	67.3	74.6
Half-cauchy	10	53.8	2.6	41.8	35.7	44.4	37.5	45.4
	20	75.6	0.6	65.5	57.3	68.4	58.8	68.7
	50	95.3	0.0	92.5	88.4	94.0	88.9	93.9
Half-normal	10	1.8	25.0	17.7	17.0	19.6	18.8	15.3
	20	1.2	34.8	24.6	22.5	28.3	24.5	23.8
	50	0.9	64.9	51.8	47.1	60.1	49.8	56.6
Log-normal(1)	10	21.8	7.2	14.2	13.0	15.5	13.8	14.9
	20	29.1	6.0	20.7	16.3	22.6	18.9	22.2
	50	42.4	5.4	32.1	29.5	36.4	32.3	35.3
Log-norm(2.4)	10	86.9	0.4	76.3	71.6	79.4	73.4	84.6
	20	99.0	0.0	97.4	95.4	98.3	95.8	99.1
	50	100.0	0.1	100.0	100.0	100.0	100.0	100.0
Uniform	10	3.2	48.5	37.9	43.0	45.5	43.6	39.0
	20	15.4	78.0	66.7	75.8	78.0	72.8	74.9
	50	83.0	98.9	96.8	99.7	99.6	98.3	99.6
Weibull (0.5)	10	68.4	0.5	52.5	45.2	55.4	48.2	66.4
	20	92.0	0.0	85.3	77.2	88.0	79.8	92.8
	50	100.0	0.4	100.0	99.6	100.0	99.8	100.0
Weibull (2.0)	10	1.1	42.6	33.2	31.4	38.9	34.8	31.8
	20	2.9	76.0	65.4	63.4	74.2	66.4	70.1
	50	23.2	99.5	98.3	98.0	99.4	98.7	99.3

TABLE 7

Power Comparisons using Method 2

(using unbiased estimates of A and B)

The table shows the percentage of 2500 samples declared significant by the statistic, when the test used was a 10% test.

Alternative distr.	Sample size	D^+	D^-	D	V	W^2	U^2	A^2
Beta (1,4)	10	4.3	16.0	11.8	11.6	13.7	11.9	12.9
	20	3.1	22.0	15.1	15.6	17.8	14.4	16.4
	50	1.0	38.6	26.9	22.8	32.1	24.7	29.1
Chi-square 1	10	39.8	2.4	23.5	22.1	26.8	25.4	31.6
	20	64.3	0.9	48.1	42.8	52.4	46.1	59.1
	50	93.8	1.0	97.3	81.5	91.6	85.7	95.0
Chi-square 4	10	2.5	23.6	16.4	16.1	20.0	16.7	18.5
	20	1.6	41.6	31.1	29.1	35.5	29.7	33.9
	50	1.7	77.9	67.7	63.3	75.7	67.8	76.0
Half-cauchy	10	53.4	2.4	42.0	36.0	45.9	40.6	48.0
	20	76.3	0.6	66.1	58.8	69.1	61.2	70.3
	50	95.1	0.0	92.5	88.4	94.3	89.6	94.2
Half-normal	10	1.5	23.9	16.7	16.5	19.4	17.0	17.8
	20	1.4	34.2	24.8	23.9	28.6	23.2	26.2
	50	0.7	64.8	51.5	46.7	61.2	50.3	59.2
Log-normal(1)	10	20.3	6.7	14.4	13.0	16.1	14.5	17.0
	20	29.8	5.9	21.3	17.9	23.3	19.4	24.1
	50	40.6	5.4	32.1	29.3	37.2	33.1	37.4
Log-norm(2.4)	10	86.6	0.3	76.9	71.9	80.4	77.5	82.9
	20	99.0	0.0	97.5	95.8	98.5	96.5	99.0
	50	100.0	0.1	100.0	100.0	100.0	100.0	100.0
Uniform	10	2.5	47.2	35.6	42.6	45.0	40.4	44.8
	20	16.7	77.6	66.8	77.4	78.4	71.2	79.2
	50	80.9	98.9	96.8	99.7	99.6	98.3	99.7
Weibull (0.5)	10	68.0	0.5	53.2	45.8	57.3	53.2	62.1
	20	92.5	0.0	86.0	78.6	88.5	82.4	91.0
	50	100.0	0.4	100.0	99.5	100.0	99.8	100.0
Weibull (2.0)	10	0.8	41.6	31.4	30.8	38.4	31.8	36.3
	20	3.4	75.8	65.5	65.0	74.6	65.2	73.2
	50	20.7	99.5	98.3	98.0	99.4	98.8	99.4

TABLE 8

Power Comparisons: regression and EDF statistics

The table shows the percentage of 2500 samples declared significant by the statistic, when the test used was a 10% test.

Alternative distr.	Sample size	T_m	T_h	W_E	W^2	A^2
Beta (1,4)	10	13.4	8.6	11.4	13.7	12.9
	20	13.2	5.6	17.3	17.8	16.4
	50	22.2	6.7	43.5	32.1	29.1
Chi-square 1	10	13.4	18.4	25.0	26.8	31.6
	20	17.8	23.0	39.0	52.4	59.1
	50	29.4	36.5	78.6	91.6	95.0
Chi-square 4	10	13.5	9.8	16.0	20.0	18.5
	20	13.2	7.5	29.2	35.5	33.9
	50	19.4	9.4	72.7	75.7	76.0
Half-cauchy	10	36.8	44.1	48.3	45.9	48.0
	20	63.0	68.1	71.6	69.1	70.3
	50	92.0	93.8	96.0	94.3	94.2
Half-normal	10	15.8	10.2	17.0	19.4	17.8
	20	19.3	9.3	29.6	28.6	26.2
	50	32.2	12.6	72.4	61.2	59.2
Log-normal(1)	10	14.4	19.0	19.1	16.1	17.0
	20	24.4	29.5	26.5	23.3	24.1
	50	42.8	49.4	47.4	37.2	37.4
Log-norm (2.4)	10	54.2	64.4	76.4	80.4	82.9
	20	81.5	86.6	96.0	98.5	99.0
	50	98.8	99.1	100.0	100.0	100.0
Uniform	10	56.0	40.2	48.0	45.0	44.8
	20	88.7	71.3	82.6	78.4	79.2
	50	100.0	99.5	99.8	99.6	99.7
Weibull (0.5)	10	30.3	40.6	54.4	57.3	62.1
	20	49.8	58.4	82.0	88.5	91.0
	50	82.5	85.8	99.4	100.0	100.0
Weibull (2.0)	10	27.4	18.8	37.8	38.4	36.3
	20	38.8	22.0	73.3	74.6	73.2
	50	71.6	48.0	99.7	99.4	99.4

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20. ABSTRACT

TESTS FOR EXPONENTIALITY WHEN ORIGIN AND SCALE
PARAMETERS ARE UNKNOWN

By

John Spinelli and
Michael A. Stephens

(empirical distribution function)

Several methods are examined for testing for the exponential distribution, when both the scale and location parameters are unknown. Percentage points are given for EDF tests, in which both parameters are estimated from the data, and the tests are compared for power with EDF tests where only the scale parameter is unknown. In addition, power studies are reported to compare these tests with other goodness-of-fit techniques for the exponential distribution, particularly those based on regression methods, and it is found that the EDF statistics W^2 and A^2 have overall good power properties.

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